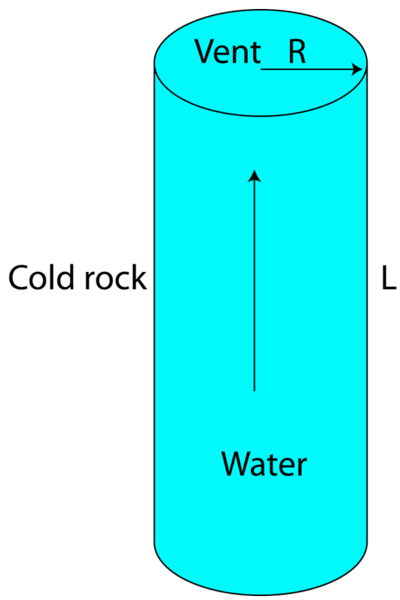
Geophysics 150: Homework 2
(Due February 05 2008)

We continue with Mars ice. The mean annual temperature is about -70°C. The volume specific heat of ice is 2×10^6 J m^{-3} K^{-1}. The thermal conductivity of ice is 2.3 W m^{-1} K^{-1}. The latent heat of freezing of water to ice is 320×10^6 J m^{-3}. The heat flow is 30 mW m^{-2}.

1. Natural hot springs in the high Arctic are often cylindrical holes. There are hints from images at water erupts this way on Mars. Volcanic vents are also cylindrical after the start of eruptions



a. The radius R of the vent sets the length scale for conduction. So the conductive heat flow away from the tube is approximately

$$q = k\frac{\Delta T}{R}$$

The total heat loss per vertical length of cylinder Q is this heat flow time the circumference. Obtain the expression. Do we need to know the radius accurately to proceed?

b. The heat loss from the total tube is QL where L is the vertical height of the tube. Use parameters for Mars ice and let the water be 0°C. What is the heat loss from the tube?

c. The temperature decreases linearly from -70°C to 0°C at depth L where the water starts up. Compute the depth L. Note that the heat loss is actually half that QL you computed in part (b) because its local value Q depends linearly on the temperature contrast between the water and the ice.

c. Assume for now that the water starts out at 1°C and cools to 0°C. What flux volume per time of water (cooling by 1 K) is needed to balance the conductive heat flux? This sets a scale for the minimum flow for continuously flowing vents to remain fully liquid.

d. More likely slush with some water can flow through the tube. Assume than flow continues until the fluid is 75% ice? What fluid flux is needed to balance the heat loss. Does it matter much whether the ice starts up at 1°C or 0°C?