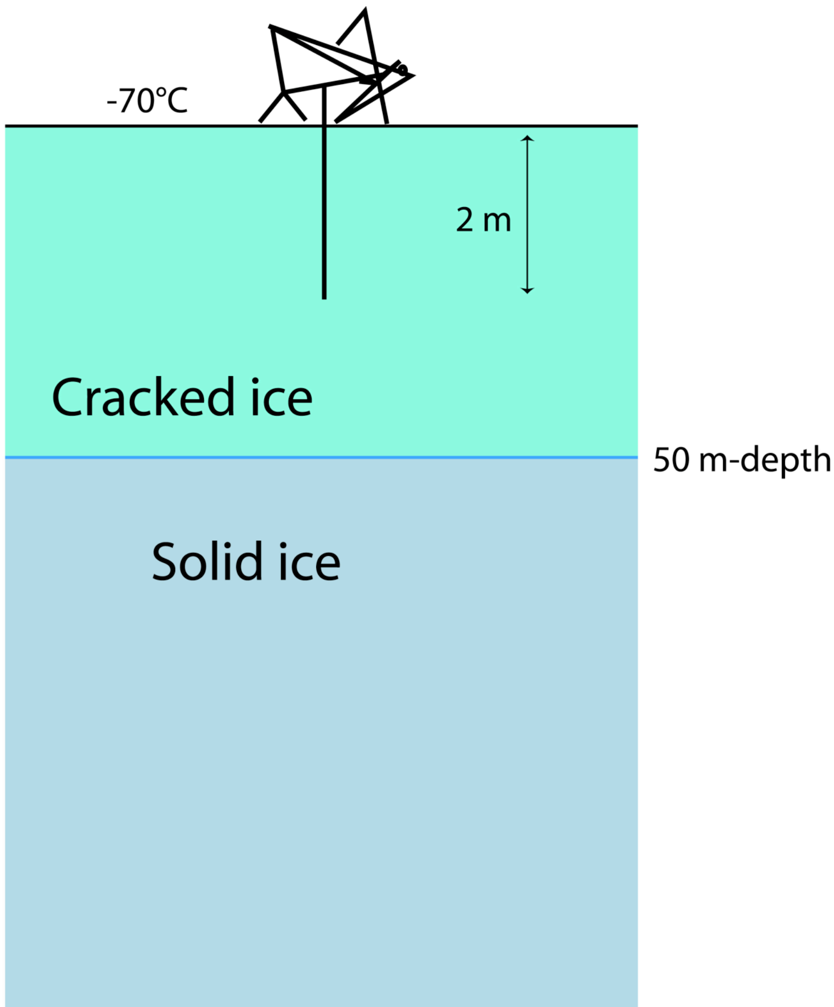
Geophysics 150: Homework 1
(Due January 29 2008)

We land a probe on the icecap of Mars. The mean annual temperature here is about -70°C. Our probe can go 2-m into the ice. The volume specific heat of ice is 2×10^6 J m^{-3} K^{-1}. The thermal conductivity of our ice is only 0.3 W m^{-1} K^{-1} because there is open pore space. The latent heat of freezing of water to ice is 320×10^6 J m^{-3}.



1. Assume steady state. We guess in analogy to the Earth that the heat flow on Mars is 30 mW m^{-2}. Compute the temperature at 2-m depth.

2. You are worried about whether you are justified in considering that the ice is in steady state.

a. There are daily temperature variations at the surface. (1 day = ~86400 s as on the Earth). Obtain thermal diffusivity and use the dimensional distance = sqrt (time * thermal diffusivity) to obtain the depth that daily variations penetrate into the ice.

b. The winter lasts about 3.5 earth months or 10^7 s. Compare this time with the scale time thickness^2 / thermal diffusivity. Does 2 m thick ice come into steady state? Conversely, will the probe sense the seasonal variations at 2 m depth.

3. We would like to find liquid brine at ~20°C to look for life.

(a) How deep would we need to drill using the surface conductivity?

(b) Assume instead that the ice is hard and pore free below the depth of 50 m. It has the thermal conductivity of pure ice 2.3 W m^{-1} K^{-1}. Compute the thermal gradient below this depth.

(c) Sketch the geotherm and compute how deep we need to go to find -20°C brines. Assume steady state and that the icecap is in fact at least this thick.

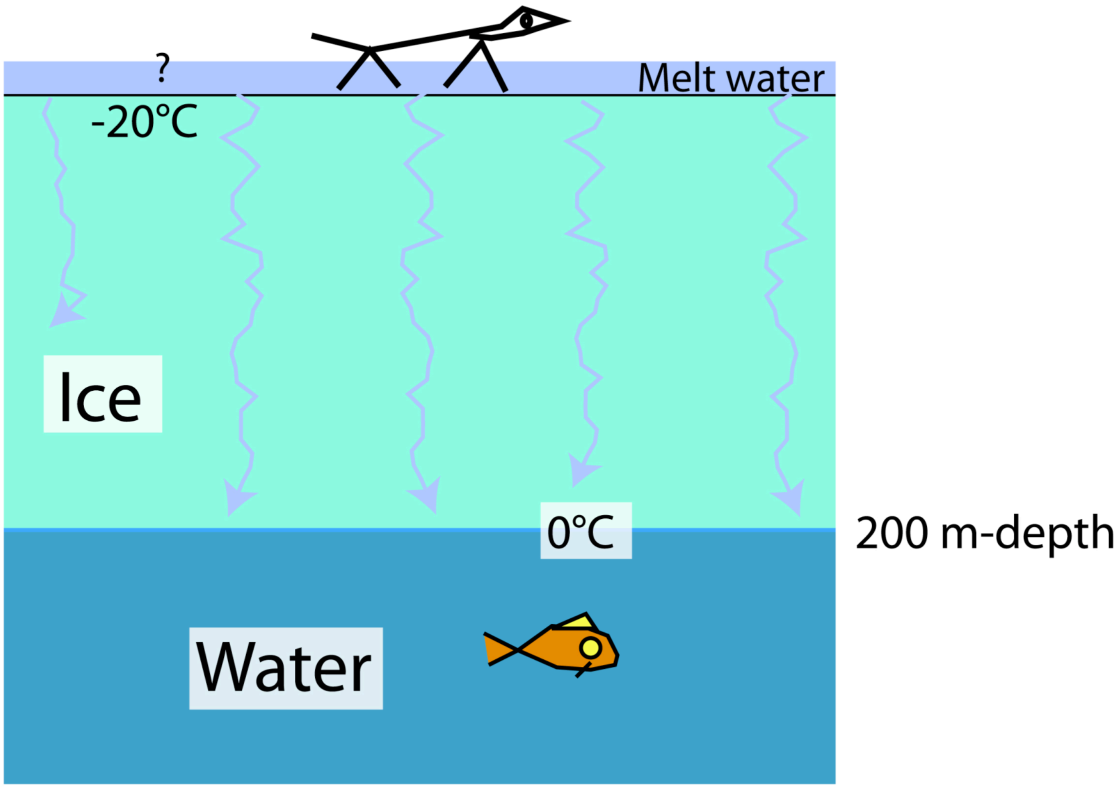
4. Our probe melts a 0.01 m cube of ice with a mass of 0.001 kg or equivalently a volume of 10^{-6} m^3. It is within the shallow cracked ice at -70°C.

a. Compute the specific heat to raise the ice to 0°C and the latent heat to melt this ice.

b. Use this information to compute an effective diffusivity for the ice that melts.

c. Obtain the scale time Z^2/κ for the 0.01 m distance with this diffusivity. Use this result and the total heat in a to find how the power (heat/time) need for the probe to actually melt the ice.

d. The solar flux in this area is 0.05 W m^{-2}. How big a solar collector would we need to get this power if it was perfectly efficient?



5. This is a related problem from Al Gore's movie. Melting occurs at the top of the ice in the summer. The melt water percolates downward into cracks. The water freezes and the latent heat warms the ice. How much (thickness of standing) water is needed to raise the 200 m of ice from its ambient temperature to 0°C assuming a conductive profile starting at -20°C at the surface? Note that the surface temperature is colder than the ocean so the geothermal gradient is reversed from our case on Mars. Assume ice is 200 m thick. This is a measure of the thickness of melt water needed to weaken the ice sheet. Thick ice sheets fail in this manner over a period of years.